Ch 3 – Understanding money management

1. nominal & effective interest rates
2. equivalence calculations using effective interest rates
3. debt management

- If payments occur more frequently than annual, how do you calculate economic equivalence?
- If interest period is other than annual, how do you calculate economic equivalence?
- How are commercial loans structured?
- How should you manage your debt?
### Nominal vs. effective interest rates

<table>
<thead>
<tr>
<th>Nominal interest rate:</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate quoted based on an annual period (APR)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effective interest rate:</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual interest earned or paid in a year (or some other time period)</td>
</tr>
</tbody>
</table>

**Example:** 18% compounded monthly
- interest rate per month: \( i = \frac{18\%}{12} = 1.5\% \)
- no. interest periods per year: \( N = 12 \)
- **borrow:** bank charges 1.5% interest each month on your unpaid balance
- **deposit:** you earn 1.5% interest each month on your remaining balance
Question: Suppose that you invest $1,000 for 1 year at 18% compounded monthly. How much interest would you earn?

\[ F = 1,000(1 + i)^N = 1,000(1 + 0.015)^{12} = 1,195.60 \]

\[ i = 0.1956 \rightarrow 19.56\% \]

= 1.5%
Effective annual interest rate (yield)

\[ i_a = (1 + \frac{r}{M})^M - 1 \]

\( r \) = nominal interest rate per year (APR)  
\( i_a \) = effective annual interest rate  
\( M \) = number of interest periods per year

18% compounded monthly \( \Rightarrow \) 1.5% per month for 12 months

\[ \Rightarrow \] 19.56% compounded annually
Practice problems

- If your credit card calculates interest based on 12.5% APR, what are your monthly interest rate & annual effective interest rate?

- If your credit card’s current outstanding balance is $2,000 & you decide to skip payments for 2 months, what would be the total balance 2 months from now?

\[
\text{monthly: } i = \frac{12.5\%}{12} = 1.0417\% \\
\text{effective annual: } i_a = (1 + 1.010417)^{12} = 13.24\% \\
\text{balance in 2 mo.: } F = $2,000(F/P, 1.0417\%,2) = $2,041.88
\]
Practice problem

Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have at the end of the year?
Practice problem

Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have at the end of the year?

(a) Interest rate per quarter:
\[ i = \frac{9\%}{4} = 2.25\% \]

(b) Annual effective interest rate:
\[ i_a = (1 + 0.0225)^4 - 1 = 9.31\% \]

(c) Balance at the end of one year (after 4 quarters)
\[ F = $10,000(F / P, 2.25\%, 4) \]
\[ = $10,000(F / P, 9.31\%, 1) \]
\[ = $10,931 \]
Brute force method of solution

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Calculation</th>
<th>Value after one year</th>
</tr>
</thead>
<tbody>
<tr>
<td>First quarter</td>
<td>base amount + interest (2.25%)</td>
<td>$10,000 + $225</td>
</tr>
<tr>
<td>Second quarter</td>
<td>= new base amount + interest (2.25%)</td>
<td>= $10,225 + $230.06</td>
</tr>
<tr>
<td>Third quarter</td>
<td>= new base amount + interest (2.25%)</td>
<td>= $10,455.06 + $235.24</td>
</tr>
<tr>
<td>Fourth quarter</td>
<td>= new base amount + interest (2.25 %) = value after one year</td>
<td>= $10,690.30 + $240.53 = $10,930.83</td>
</tr>
</tbody>
</table>

Effective annual interest rate (9% compounded quarterly)
Example 3.4: Calculating auto loan payments

Given:
Invoice price = $21,599
Sales tax at 4% = $21,599 (0.04) = $863.96
Dealer’s freight = $21,599 (0.01) = $215.99
Total purchase price = $22,678.95
Down payment = $2,678.95
Dealer’s interest rate = 8.5% APR, monthly compounding
Length of financing = 48 months

Find: monthly payment
Solution: Payment period = Interest period

Given:
- \( P = $20,000 \)
- \( r = 8.5\% \text{ per year} \)
- \( K = 12 \text{ payments per year} \)
- \( N = 48 \text{ payment periods} \)

Find \( A \):

- \( M = 12 \text{ compounding periods per year} \)
- \( i = \frac{r}{M} = \frac{8.5\%}{12} = 0.7083\% \text{ per month} \)
- \( N = (12)(4) = 48 \text{ months, or payment periods} \)
- \( A = $20,000\left(\frac{A}{P}, 0.7083\%, 48\right) = $492.97 \)
Suppose you want to pay off the remaining loan in lump sum right after making the 25th payment. How much would this lump be?

\[ P = \$492.97 \times (P/A, 0.7083\%, 23) = \$10,428.96 \]
Practice problem

You have a habit of drinking a cup of Starbucks coffee ($2.00 a cup) on the way to work every morning for 30 years. If you put the money in the bank for the same period, how much would you have, assuming your accounts earns 5% interest compounded daily.

NOTE: Assume you drink a cup of coffee every day including weekends.
Practice problem

You have a habit of drinking a cup of Starbuck coffee ($2.00 a cup) on the way to work every morning for 30 years. If you put the money in the bank for the same period, how much would you have, assuming your accounts earns 5% interest compounded daily.

NOTE: Assume you drink a cup of coffee every day including weekends.

\[
i = \frac{5\%}{365} = 0.0137\% \text{ per day}
\]

\[
N = 30 \times 365 = 10,950 \text{ days}
\]

\[
F = $2(F/A, 0.0137\%, 10950)
\]

\[
= $50,831
\]
Effective interest rate per payment period
Effective interest rate per payment period

\[ i = \left[ 1 + \frac{r}{CK} \right]^C - 1 \]

- \( C \) = number of interest periods per payment period
- \( K \) = number of payment periods per year
- \( CK \) = total number of interest periods per year, or \( M \)
- \( \frac{r}{K} \) = nominal interest rate per payment period
12% compounded monthly, quarterly payments

One year

1st 2nd 3rd 4th

1% 1% 1%

3.03% 3.03%

12.68%

Effective interest rate per quarter

\[ i = (1 + 0.01)^3 - 1 = 3.030\% \]

Effective annual interest rate

\[ i_a = (1 + 0.01)^{12} - 1 = 12.68\% \]

\[ i_a = (1 + 0.03030)^4 - 1 = 12.68\% \]
Effective interest rate per payment period with continuous compounding

\[ i = \left(1 + \frac{r}{CK}\right)^C - 1 \]

\( CK = \) number of compounding periods per year

for continuous compounding: \( C \to \infty \)

\[ i = \lim_{C \to \infty} \left(1 + \frac{r}{CK}\right)^C - 1 = (e^r)^{1/K} - 1 \]
Example: effective interest rate per quarter

\[ i = \left(1 + \frac{r}{CK}\right)^C - 1 \]

\[ r = 0.08 \]

\[ K = 4 \text{ payments per year} \]

<table>
<thead>
<tr>
<th>compound quarterly</th>
<th>compound monthly</th>
<th>compound weekly</th>
<th>compound continuously</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = 1 )</td>
<td>( C = 3 )</td>
<td>( C = 13 )</td>
<td>( C \to \infty )</td>
</tr>
<tr>
<td>( M = 4 )</td>
<td>( M = 12 )</td>
<td>( M = 52 )</td>
<td></td>
</tr>
<tr>
<td>( i = \left[1 + \frac{0.08}{4}\right]^1 - 1 )</td>
<td>( i = \left[1 + \frac{0.08}{12}\right]^3 - 1 )</td>
<td>( i = \left[1 + \frac{0.08}{52}\right]^{13} - 1 )</td>
<td>( i = e^{0.02} - 1 )</td>
</tr>
<tr>
<td>2.000% per qtr</td>
<td>2.013% per qtr</td>
<td>2.0186% per qtr</td>
<td>2.0201% per qtr</td>
</tr>
</tbody>
</table>
Example 3.5 – Discrete case: quarterly deposits with monthly compounding

Suppose you make equal quarterly deposits of $1,000 into a fund that pays interest at 12% compounded monthly. Find the balance at the end of year 3.

- \( M = 12 \) compounding periods/year
- \( K = 4 \) payment periods/year
- \( C = 3 \) interest periods per quarter

\[
i = \left[ 1 + \frac{0.12}{(3)(4)} \right]^3 - 1 = 3.030\%\]

\[
N = 4(3) = 12
\]

\[
F = $1,000 \cdot (F/A, 3.030\%, 12) = $14,216.24
\]
Continuous case: Quarterly deposits with continuous compounding

\[ F = ? \]

\begin{align*}
K &= 4 \text{ payment periods/year} \\
C &= \infty \text{ interest periods per quarter} \\
i &= e^{0.12/4} - 1 \\
&= 3.045\text{\% per quarter} \\
N &= 4(3) = 12 \\
F &= $1,000 \ (F/A, 3.045\% , 12) = $14,228.37
\end{align*}
Practice problem

A series of equal quarterly payments of $5,000 for 10 years is equivalent to what present amount at an interest rate of 9% compounded

a. quarterly
b. monthly
c. continuously
Quarterly

Payment period: Quarterly
Interest Period: Quarterly

A = $5,000

\[ i = \frac{9\%}{4} = 2.25\% \text{ per quarter} \]

\[ N = 40 \text{ quarters} \]

\[ P = $5,000(P/A, 2.25\%, 40) \]
\[ = $130,968 \]
Monthly

Payment period: Quarterly
Interest Period: Monthly

\[ A = \$5,000 \]

\[ i = \frac{9\%}{12} = 0.75\% \text{ per month} \]

\[ i_p = (1 + 0.0075)^3 = 2.267\% \text{ per quarter} \]

\[ N = 40 \text{ quarters} \]

\[ P = \$5,000 \frac{P}{A, 2.267\%, 40} \]

\[ = \$130,586 \]
Continuously

Payment period: Quarterly
Interest period: Continuously

\[ i = e^{0.09/4} - 1 = 2.276\% \text{ per quarter} \]

\[ N = 40 \text{ quarters} \]

\[ P = $5,000(P \/ A, 2.276\%, 40) \]

\[ = $130,384 \]
Example 3.7 – Loan repayment schedule

For a $5,000 loan repaid over 2 years at 12% develop the monthly loan repayment schedule showing interest & principal for each period.

\[
A = $5,000 \times (A/P, 1\%, 24) = $235.37
\]

Consider the 7th payment ($235.37)

a. How much is interest?

b. What is the amount of principal payment?
Solution

How much is interest?
What is the amount of principal payment?

Outstanding balance at the end of period 6:
(Note: 18 outstanding payments)
\[ B_6 = \$235.37 \left( \frac{P}{A,1\%,18} \right) = \$3,859.66 \]

Interest payment for period 7:
\[ IP_7 = \$3,859.66 \times 0.01 = \$38.60 \]

Principal payment for period 7:
\[ PP_7 = \$235.37 - \$38.60 = \$196.77 \]

Note: \[ IP_7 + PP_7 = \$235.37 \]
### Example 3.7 Loan Repayment Schedule

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
<td><strong>Example 3.7 Loan Repayment Schedule</strong></td>
<td></td>
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<tr>
<td>5</td>
<td>Contract amount</td>
<td>$5,000.00</td>
<td>Total payment</td>
<td>$5,648.82</td>
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<tr>
<td>6</td>
<td>Contract period</td>
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<td>Total interest</td>
<td>$648.82</td>
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<td>7</td>
<td>APR (%)</td>
<td>12</td>
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<td>Monthly Payment</td>
<td>($235.37)</td>
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<tr>
<td>10</td>
<td>Payment No.</td>
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<td>Interest payment</td>
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<td>31</td>
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### Example 3.9 buying versus lease decision

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<th></th>
<th>debt financing</th>
<th>Lease financing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price</strong></td>
<td>$14,695</td>
<td>$14,695</td>
</tr>
<tr>
<td>Down payment</td>
<td>$2,000</td>
<td>0</td>
</tr>
<tr>
<td>APR (%)</td>
<td>3.6%</td>
<td></td>
</tr>
<tr>
<td>Monthly payment</td>
<td>$372.55</td>
<td>$236.45</td>
</tr>
<tr>
<td>Length</td>
<td>36 months</td>
<td>36 months</td>
</tr>
<tr>
<td>Fees</td>
<td></td>
<td>$495</td>
</tr>
<tr>
<td>Cash due at lease end</td>
<td></td>
<td>$300</td>
</tr>
<tr>
<td>Purchase option at lease end</td>
<td></td>
<td>$8,673.10</td>
</tr>
<tr>
<td>Cash due at signing</td>
<td>$2,000</td>
<td>$731.45</td>
</tr>
</tbody>
</table>
Accounting data - buying vs. lease

Cash outlay for buying: $25,886

- Down payment: $2,100
- Car Loan at 8.5%
  (48 payments of $466): $22,368
- Sales tax (at 6.75%): $1,418

Cash outlay for leasing: $15,771

- Lease (48 payments of $299): $14,352
- Sales tax (at 6.75%): $969
- Document fee: $450
- Refundable security deposit (not included in total): $300
Which interest rate to use to compare these options?
Your earning interest rate = 6%

Debt financing:

\[ P_{\text{debt}} = \$2,000 + \$372.55(P/A, 0.5\%, 36) - \$8,673.10(P/F, 0.5\%, 36) = \$6,998.47 \]

Lease financing:

\[ P_{\text{lease}} = \$495 + \$236.45 + \$236.45(P/A, 0.5\%, 35) + \$300(P/F, 0.5\%, 36) = \$8,556.90 \]
Summary

- Financial institutions often quote interest rate based on an APR.
- In all financial analysis, we need to convert the APR into an appropriate effective interest rate based on a payment period.
- When payment period and interest period differ, calculate an effective interest rate that covers the payment period. Then use the appropriate interest formulas to determine the equivalent values.