# Ch 3 – Understanding money management

1. nominal & effective interest rates
2. equivalence calculations using effective interest rates
3. debt management

- If payments occur more frequently than annual, how do you calculate economic equivalence?
- If interest period is other than annual, how do you calculate economic equivalence?
- How are commercial loans structured?
- How should you manage your debt?

# Nominal vs. effective interest rates

Nominal interest rate:
rate quoted based on an annual period (APR)

Effective interest rate:
actual interest earned or paid in a year (or some other time period)

Example: 18% compounded monthly

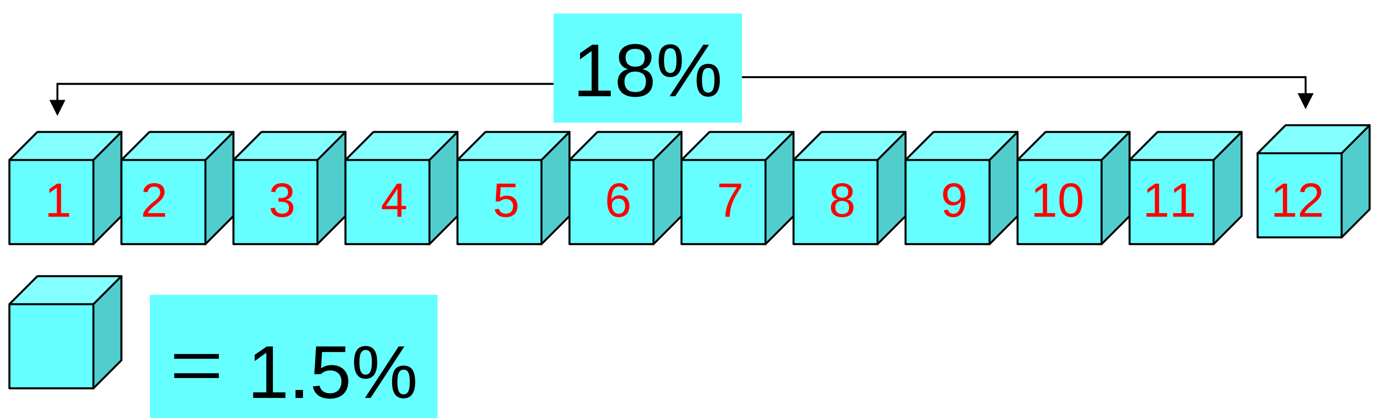
- interest rate per month: $i$ = 18%/12 = 1.5%
- no. interest periods per year: N = 12
- borrow: bank charges 1.5% interest each month on your unpaid balance
- deposit: you earn 1.5% interest each month on your remaining balance

# 18% compounded monthly

Question: Suppose that you invest $1,000 for 1 year at 18% compounded monthly. How much interest would you earn?

$$F = \$1,000(1 + i)^N = \$1,000(1 + 0.015)^{12} = \$1,195.60$$

$$i = 0.1956 \rightarrow 19.56\%$$



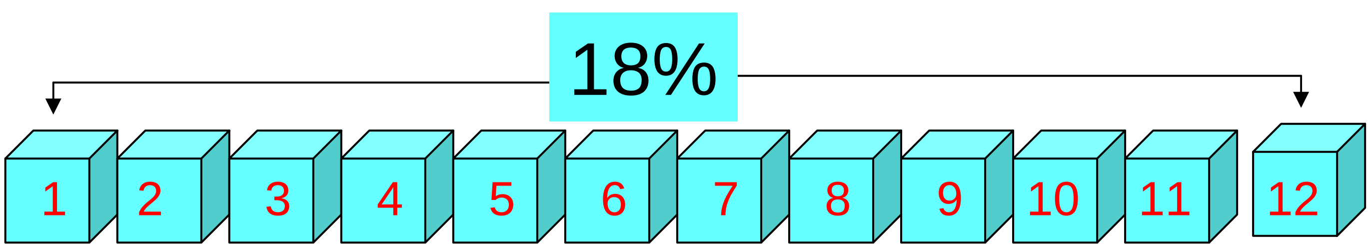
# Effective annual interest rate (yield)

$$i_a = (1 + r/M)^M - 1$$

*r* = nominal interest rate per year (APR)

$i_a$ = effective annual interest rate

*M* = number of interest periods per year




18% compounded monthly ➔ 1.5% per month for 12 months

=

19.56 % compounded annually

# Practice problems

- If your credit card calculates interest based on 12.5% APR, what are your monthly interest rate & annual effective interest rate?
- If your credit card's current outstanding balance is $2,000 & you decide to skip payments for 2 months, what would be the total balance 2 months from now?

monthly: $i = \dfrac{12.5\%}{12} = 1.0417\%$

effective annual: $i_a = (1 + 1.010417)^{12} = 13.24\%$

balance in 2 mo.: F = $2,000(F/P, 1.0417%,2) = $2,041.88

# Practice problem

Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have at the end of the year?

# Practice problem

Suppose your savings account pays 9% interest compounded quarterly. If you deposit \$10,000 for one year, how much would you have at the end of the year?

(a) Interest rate per quarter:

$$i = \frac{9\%}{4} = 2.25\%$$

(b) Annual effective interest rate:

$$i_a = (1 + 0.0225)^4 - 1 = 9.31\%$$

(c) Balance at the end of one year (after 4 quarters)

$$F = \$10,000(F/P, 2.25\%, 4)$$

$$= \$10,000(F/P, 9.31\%, 1)$$

$$= \$10,931$$

# Brute force method of solution

| | | |
|---|---|---|
| First quarter | **base amount**<br>**+ interest (2.25%)** | **$10,000**<br>**+ $225** |
| Second quarter | **= new base amount**<br>**+ Interest (2.25%)** | **= $10,225**<br>**+$230.06** |
| Third quarter | **= new base amount**<br>**+ Interest (2.25%)** | **= $10,455.06**<br>**+$235.24** |
| Fourth quarter | **= new base amount**<br>**+ interest (2.25 %)**<br>**= value after one year** | **= $10,690.30**<br>**+ $240.53**<br>**= $10,930.83** |

Effective annual interest rate (9% compounded quarterly)

# Example 3.4: Calculating auto loan payments

Given:

Invoice price = $21,599

Sales tax at 4% = $21,599 (0.04) = $863.96

Dealer’s freight = $21,599 (0.01) = $215.99

Total purchase price = $22,678.95

Down payment = $2,678.95

Dealer’s interest rate = 8.5% APR, monthly compounding

Length of financing = 48 months

Find: monthly payment

# Solution: Payment period = Interest period


Given: *P* = $20,000, *r* = 8.5% per year

*K* = 12 payments per year

*N* = 48 payment periods

Find *A:*

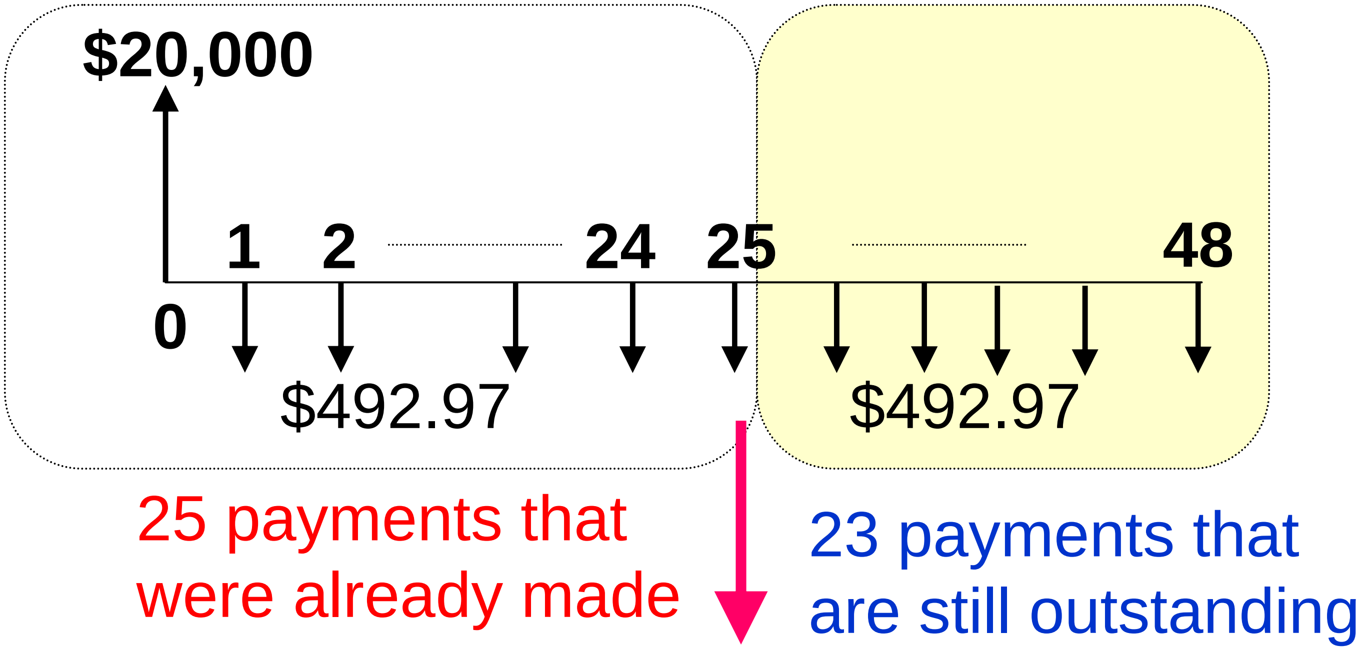
*M = 12 compounding periods per year:*

*i* = *r*/*M* = 8.5%/12 = 0.7083% per month

*N* = (12)(4) = 48 months, or payment periods

*A* = $20,000(*A*/*P*, 0.7083%,48) = $492.97

Suppose you want to pay off the remaining loan in lump sum right after making the 25th payment. How much would this lump be?




$P$ = \$492.97 ($P/A$, 0.7083%, 23) = \$10,428.96

# Practice problem

You have a habit of drinking a cup of Starbuck coffee ($2.00 a cup) on the way to work every morning for 30 years. If you put the money in the bank for the same period, how much would you have, assuming your accounts earns 5% interest compounded daily.

NOTE: Assume you drink a cup of coffee every day including weekends.

# Practice problem

You have a habit of drinking a cup of Starbuck coffee ($2.00 a cup) on the way to work every morning for 30 years. If you put the money in the bank for the same period, how much would you have, assuming your accounts earns 5% interest compounded <u>daily</u>.

NOTE: Assume you drink a cup of coffee every day including weekends.

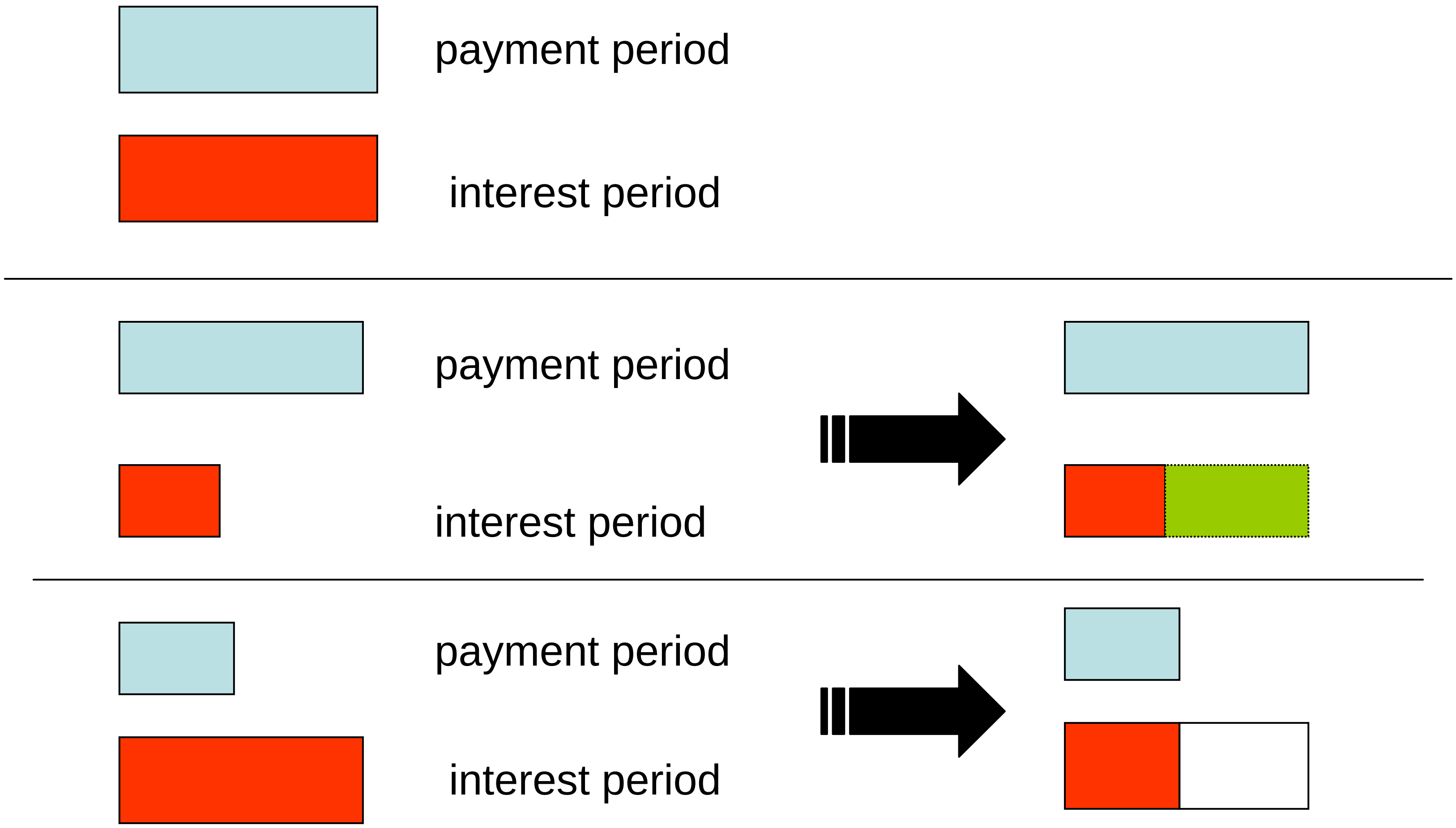
$$i = \frac{5\%}{365} = 0.0137\% \text{ per day}$$

$$N = 30 \times 365 = 10,950 \text{ days}$$

$$F = \$2(F/A, 0.0137\%, 10950)$$

$$= \$50,831$$

# Effective interest rate per payment period



# Effective interest rate per payment period

$$i = [1 + r/CK]^{C} - 1$$

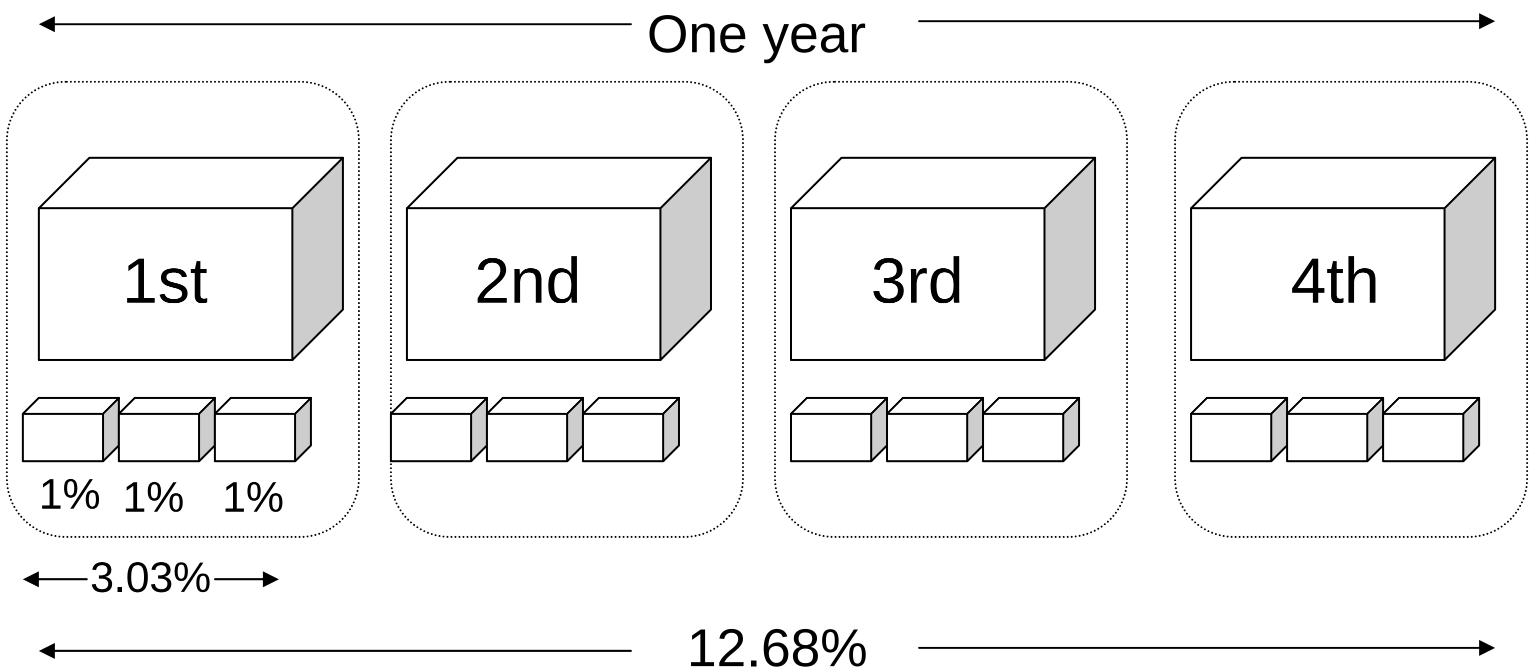
C = number of interest periods per payment period

K = number of payment periods per year

CK = total number of interest periods per year, or M

r / K = nominal interest rate per payment period

# 12% compounded monthly, quarterly payments




Effective interest rate per quarter

$$i = (1 + 0.01)^3 - 1 = 3.030\ \%$$

Effective annual interest rate

$$i_a = (1 + 0.01)^{12} - 1 = 12.68\%$$

$$i_a = (1 + 0.03030)^4 - 1 = 12.68\%$$

# Effective interest rate per payment period w/ continuous compounding

$$i = [(1 + r/CK)^C - 1]$$

*CK* = number of compounding periods per year

for continuous compounding: $C \rightarrow \infty$

$$i = \lim[(1 + r/CK)^C - 1] = (e^r)^{1/K} - 1$$

# Example: effective interest rate per quarter

$$i = [(1 + r/CK)^C – 1]$$

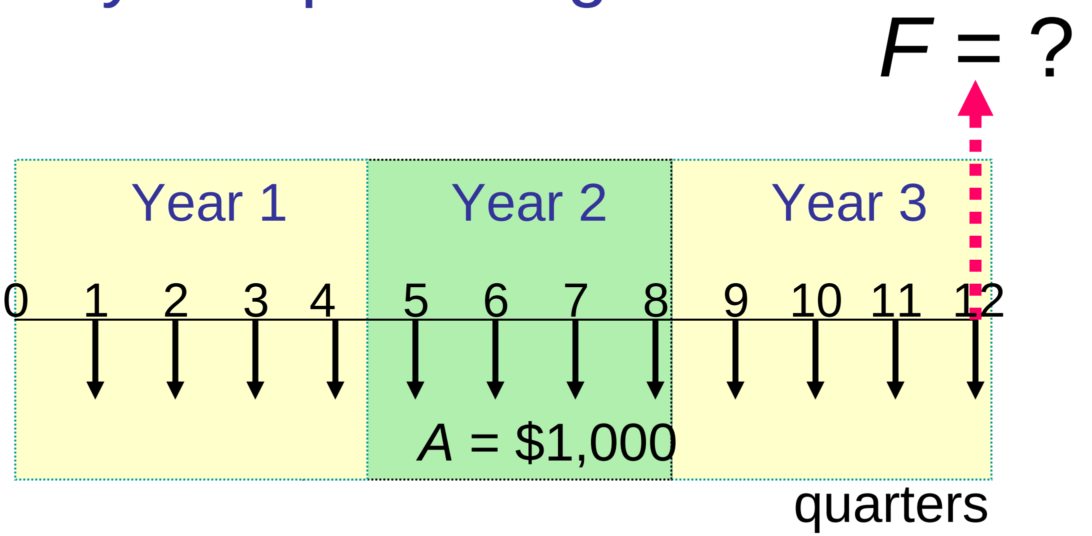
*r = 0.08*

*K* = 4 payments per year

| compound quarterly | compound monthly | compound weekly | compound continuously |
|---|---|---|---|
| *C* = 1<br>*M* = 4 | *C* = 3<br>*M* = 12 | *C* = 13<br>*M* = 52 | $C \to \infty$ |
| $i = [1 + 0.08/4]^1 - 1$ | $i = [1 + 0.08/12]^3 - 1$ | $i = [1 + 0.08/52]^{13} - 1$ | $i = e^{0.02} - 1$ |
| 2.000% per qtr | 2.013% per qtr | 2.0186% per qtr | 2.0201% per qtr |

# Example 3.5 – Discrete case: quarterly deposits with monthly compounding

Suppose you make equal quarterly deposits of $1,000 into a fund that pays interest at 12% compounded monthly. Find the balance at the end of year 3.

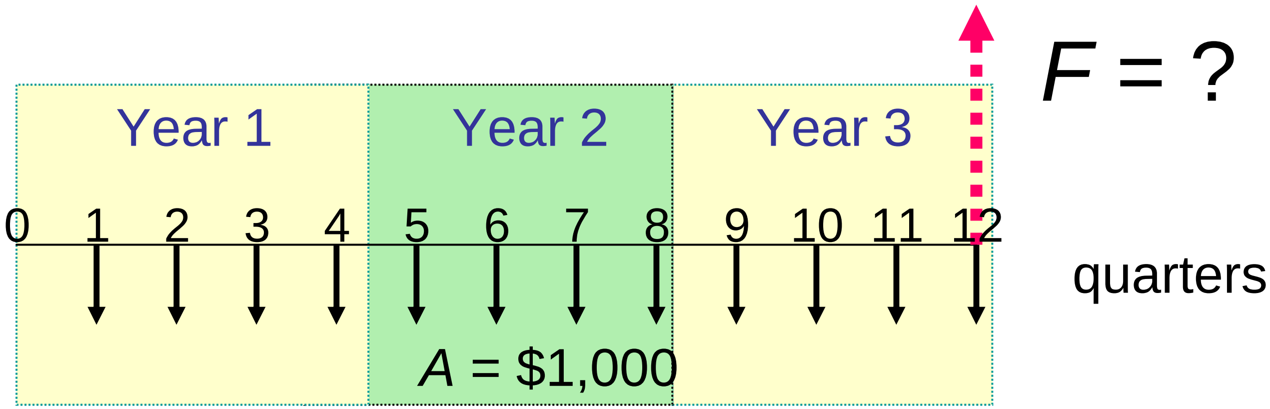



*M* = 12 compounding periods/year
*K* = 4 payment periods/year
*C* = 3 interest periods per quarter

$$i = [1 + 0.12/(3)(4)]^3 - 1 = 3.030\%$$

*N* = 4(3) = 12
*F* = $1,000 (*F*/*A*, 3.030%, 12) = $14,216.24

# Continuous case: Quarterly deposits with continuous compounding




*K* = 4 payment periods/year
*C* = ∞ interest periods per quarter

$$i = e^{0.12/4} - 1$$

$$= 3.045\% \text{ per quarter}$$

*N* = 4(3) = 12
*F* = $1,000 (*F/A*, 3.045%, 12) = $14,228.37

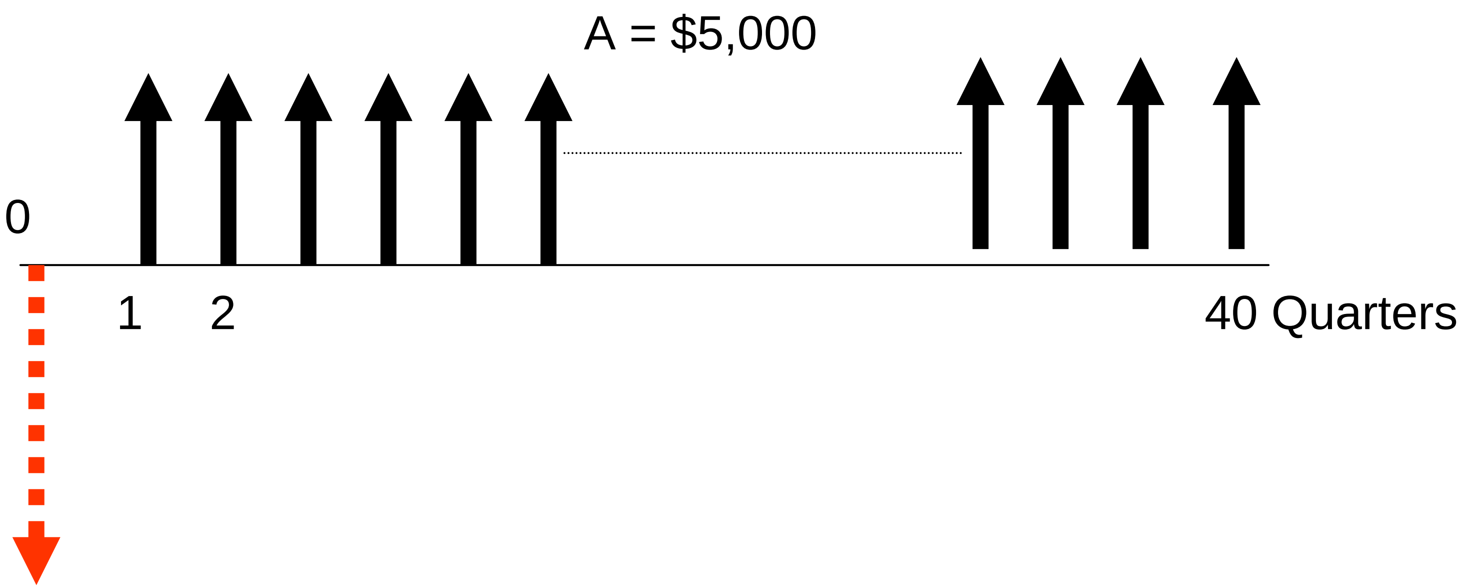
# Practice problem

A series of equal quarterly payments of $5,000 for 10 years is equivalent to what present amount at an interest rate of 9% compounded

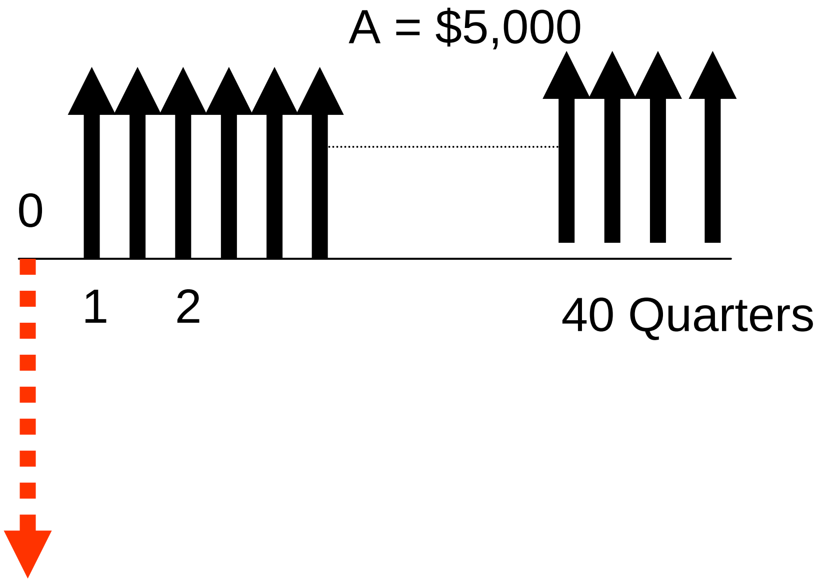
a. quarterly

b. monthly

c. continuously



# Quarterly




Payment period : Quarterly

Interest Period: Quarterly

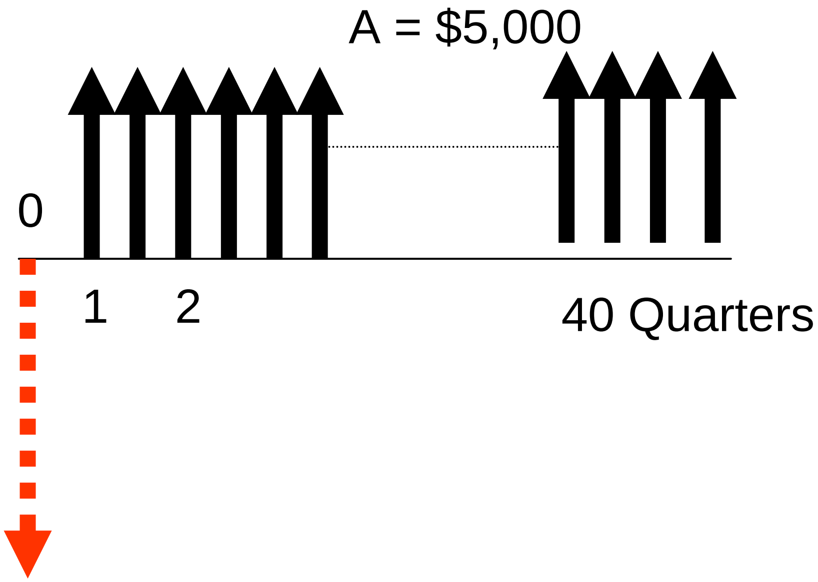
$$i = \frac{9\%}{4} = 2.25\% \text{ per quarter}$$

$$N = 40 \text{ quarters}$$

$$P = \$5{,}000(P/A, 2.25\%, 40)$$

$$= \$130{,}968$$

# Monthly




Payment period : Quarterly

Interest Period: Monthly

$$i = \frac{9\%}{12} = 0.75\% \text{ per month}$$

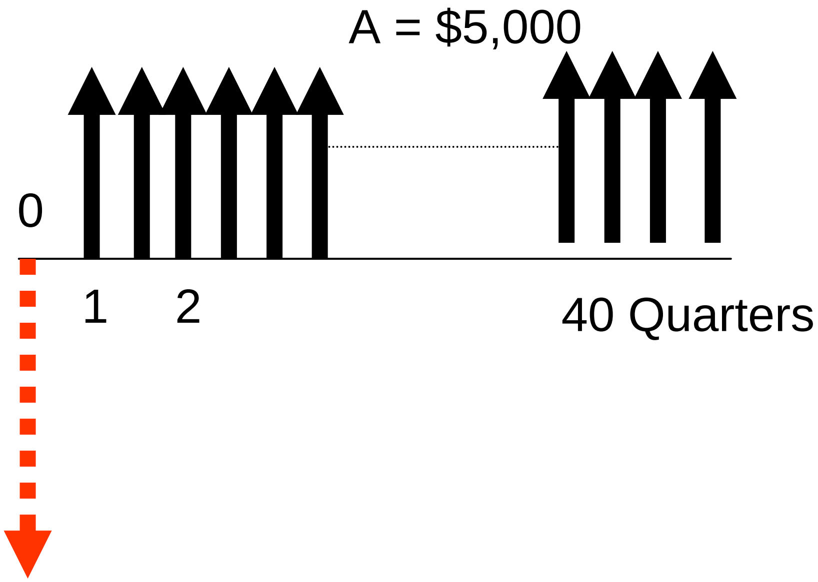
$$i_p = (1 + 0.0075)^3 = 2.267\% \text{ per quarter}$$

$$N = 40 \text{ quarters}$$

$$P = \$5,000(P/A, 2.267\%, 40)$$

$$= \$130,586$$

# Continuously




Payment period : Quarterly

Interest period: Continuously

$$i = e^{0.09/4} - 1 = 2.276\% \text{ per quarter}$$

$$N = 40 \text{ quarters}$$

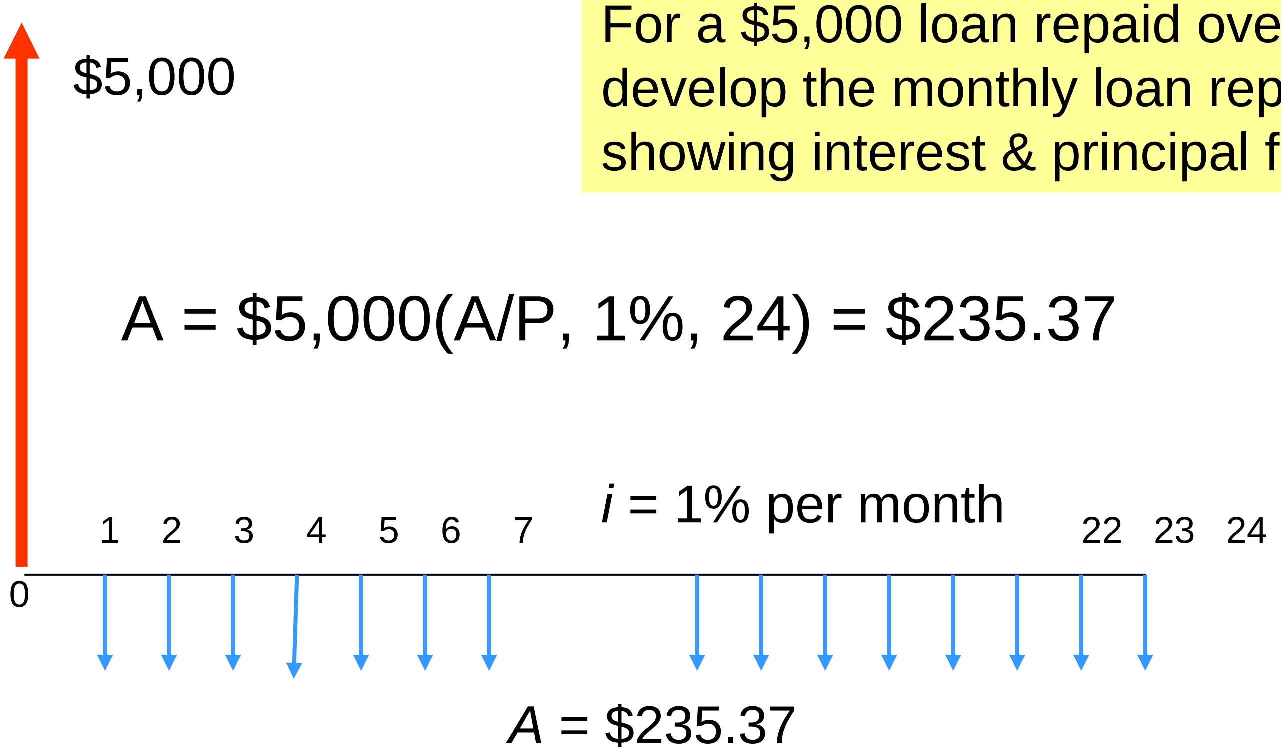
$$P = \$5,000(P/A, 2.276\%, 40)$$

$$= \$130,384$$

# Example 3.7 – Loan repayment schedule

For a \$5,000 loan repaid over 2 years at 12% develop the monthly loan repayment schedule showing interest & principal for each period.

$$A = \$5{,}000(A/P, 1\%, 24) = \$235.37$$




Consider the 7th payment (\$235.37)

a. How much is interest?

b. What is the amount of principal payment?

# Solution

How much is interest?

What is the amount of principal payment?

WOutstanding balance at the end of period 6:

(Note: 18 outstanding payments)

$$B_6 = \$235.37(P/A, 1\%, 18) = \$3,859.66$$

WInterest payment for period 7:

$$IP_7 = \$3,859.66(0.01) = \$38.60$$

WPrincipal payment for period 7:

$$PP_7 = \$235.37 - \$38.60 = \$196.77$$

Note: $IP_7 + PP_7 = \$235.37$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | Example 3.7 Loan Repayment Schedule | | | | | | |
| 4 | | | | | | | |
| 5 | Contract amount | $ 5,000.00 | | Total payment | | $ 5,648.82 | |
| 6 | Contract period | 24 | | Total interest | | $648.82 | |
| 7 | APR (%) | 12 | | | | | |
| 8 | Monthly Payment | ($235.37) | | | | | |
| 9 | | | | | | | |
| 10 | | Payment No. | Payment Size | Principal Payment | Interest payment | Loan Balance | |
| 11 | | 1 | ($235.37) | ($185.37) | ($50.00) | $4,814.63 | |
| 12 | | 2 | ($235.37) | ($187.22) | ($48.15) | $4,627.41 | |
| 13 | | 3 | ($235.37) | ($189.09) | ($46.27) | $4,438.32 | |
| 14 | | 4 | ($235.37) | ($190.98) | ($44.38) | $4,247.33 | |
| 15 | | 5 | ($235.37) | ($192.89) | ($42.47) | $4,054.44 | |
| 16 | | 6 | ($235.37) | ($194.82) | ($40.54) | $3,859.62 | |
| 17 | | 7 | ($235.37) | ($196.77) | ($38.60) | $3,662.85 | |
| 18 | | 8 | ($235.37) | ($198.74) | ($36.63) | $3,464.11 | |
| 19 | | 9 | ($235.37) | ($200.73) | ($34.64) | $3,263.38 | |
| 20 | | 10 | ($235.37) | ($202.73) | ($32.63) | $3,060.65 | |
| 21 | | 11 | ($235.37) | ($204.76) | ($30.61) | $2,855.89 | |
| 22 | | 12 | ($235.37) | ($206.81) | ($28.56) | $2,649.08 | |
| 23 | | 13 | ($235.37) | ($208.88) | ($26.49) | $2,440.20 | |
| 24 | | 14 | ($235.37) | ($210.97) | ($24.40) | $2,229.24 | |
| 25 | | 15 | ($235.37) | ($213.08) | ($22.29) | $2,016.16 | |
| 26 | | 16 | ($235.37) | ($215.21) | ($20.16) | $1,800.96 | |
| 27 | | 17 | ($235.37) | ($217.36) | ($18.01) | $1,583.60 | |
| 28 | | 18 | ($235.37) | ($219.53) | ($15.84) | $1,364.07 | |
| 29 | | 19 | ($235.37) | ($221.73) | ($13.64) | $1,142.34 | |
| 30 | | 20 | ($235.37) | ($223.94) | ($11.42) | $918.40 | |
| 31 | | 21 | ($235.37) | ($226.18) | ($9.18) | $692.21 | |
| 32 | | 22 | ($235.37) | ($228.45) | ($6.92) | $463.77 | |
| 33 | | 23 | ($235.37) | ($230.73) | ($4.64) | $233.04 | |
| 34 | | 24 | ($235.37) | ($233.04) | ($2.33) | $0.00 | |
| 35 | | | | | | | |

# Example 3.9 buying versus lease decision

| | debt financing | Lease financing |
|---|---|---|
| Price | $14,695 | $14,695 |
| Down payment | $2,000 | 0 |
| APR (%) | 3.6% | |
| Monthly payment | $372.55 | $236.45 |
| Length | 36 months | 36 months |
| Fees | | $495 |
| Cash due at lease end | | $300 |
| Purchase option at lease end | | $8.673.10 |
| Cash due at signing | $2,000 | $731.45 |

# Accounting data - buying vs. lease

Cash outlay for buying : $25,886

Down payment: $2,100

Car Loan at 8.5%

(48 payments of $466): $22,368

Sales tax (at 6.75%): $1,418

Cash outlay for leasing : $15,771

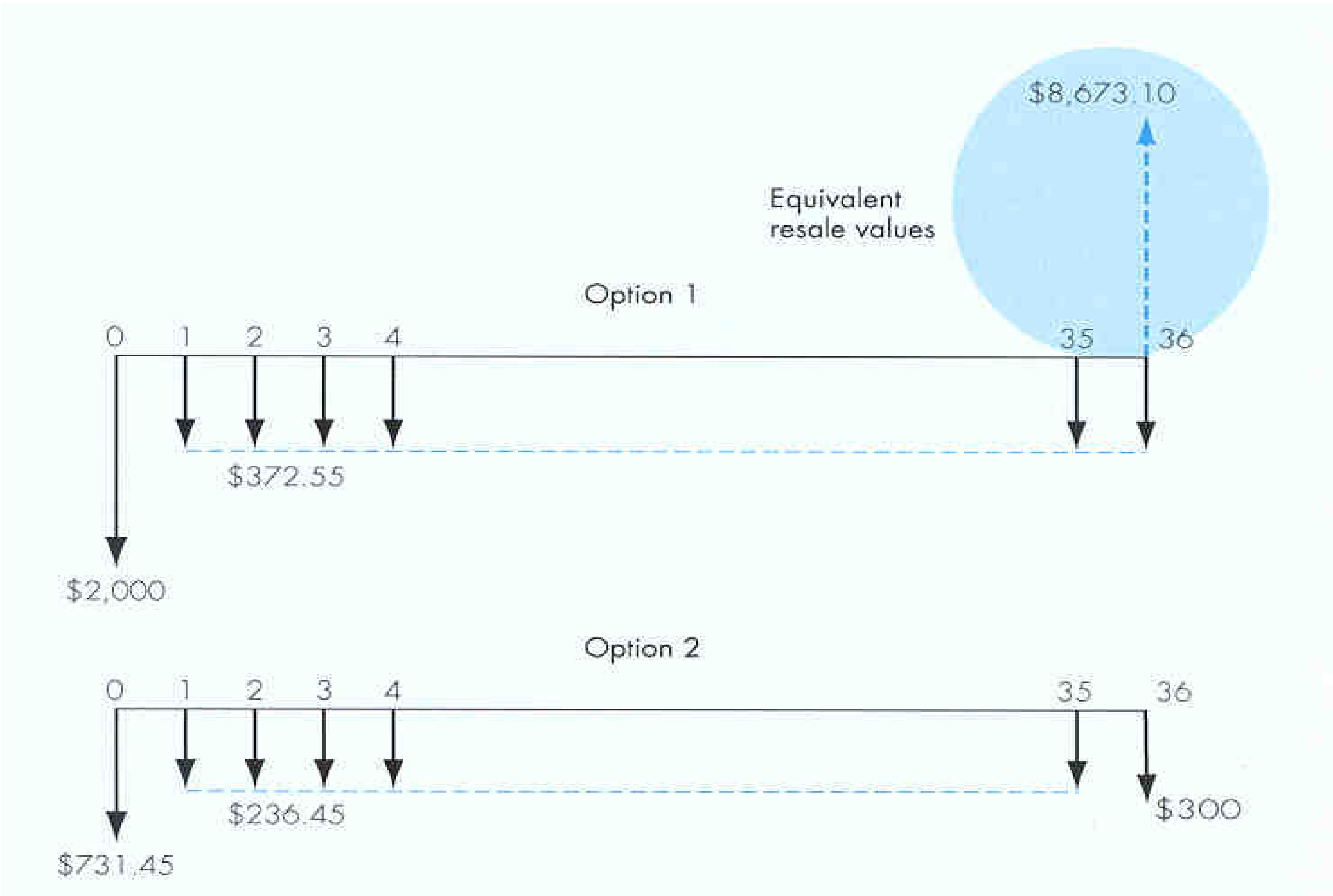
Lease (48 payments of $299) : $14,352

Sales tax (at 6.75%): $969

Document fee: $450

Refundable security deposit (not included in total) : $300

# Which interest rate to use to compare these options?



# Your earning interest rate = 6%

Debt financing:

$$P_{debt} = \$2,000 + \$372.55(P/A, 0.5\%, 36) - \$8,673.10(P/F, 0.5\%, 36) = \$6,998.47$$

Lease financing:

$$P_{lease} = \$495 + \$236.45 + \$236.45(P/A, 0.5\%, 35) + \$300(P/F, 0.5\%, 36) = \$8,556.90$$

# Summary

- Financial institutions often quote interest rate based on an APR.
- In all financial analysis, we need to convert the APR into an appropriate effective interest rate based on a payment period.
- When payment period and interest period differ, calculate an effective interest rate that covers the payment period. Then use the appropriate interest formulas to determine the equivalent values