Proceedings of Eurotherm78 - Computational Thermal Radiation in Participating Media II
5-7 April 2006, Poitiers, France

Modeling Absorption TRI In Optically Thick Eddies

by MODEST M. F.(*) and MEHTA R. S.

(*) Departmentment of Mechanical and Nuclear Engineering.
The Pennsylvania State University
University Park, PA 16802
USA
email: mfm@michelle.mne.psu.edu

Abstract
Most of modelling work to date on turbulence–radiation interactions (TRI) has been based on the so called optically thin fluctuation assumption (OTFA), which assumes that fluctuations of local properties are uncorrelated from turbulent fluctuations of the local radiative intensity. This correlation between absorption coefficient and radiative intensity, neglected by OTFA is commonly termed "absorption TRI." A model for absorption TRI is proposed for optically thick eddies, for which the OTFA is highly questionable. For spectral regions with large absorption coefficient κ, for which non-negligible absorption TRI can be expected, the diffusion approximation is known to be accurate for radiation calculations, and this method is used to extract the absorption TRI. The always important emission TRI are modeled accurately using a hybrid finite volume/PDF method. As the diffusion approximation requires the evaluation of spatial gradients, thin plate smoothing splines in multiple dimensions are used to filter out statistical noise to obtain the needed gradients. Results show that absorption TRI are very important in the active flame sheet region in optically thick spectral regions, although the total contribution to the overall radiative losses may be small for nongray gases with relatively small absorption coefficients across most of the spectrum.

1. Introduction

In many practical turbulent reactive flows, turbulent fluctuations of temperature and radiative properties have a strong influence on radiative heat transfer rates and their interactions must be considered. This requires the solution of many coupled partial differential equations, which are generally Reynolds or Favre averaged. Many "unclosed" terms appear in these equations as a result of the averaging procedure and need to be modeled. Radiative transfer appears as a radiative source term in the overall energy equation and the corresponding "unclosed" terms are a result of the TRI [1]. The importance of the interaction between radiation and turbulence is now a well-acknowledged phenomenon and significant amount of research effort has been directed toward this area. Song and Viskanta [2] investigated a turbulent premixed flame inside a two-dimensional combustor and considered TRI in a coupled way using an assumed clipped Gaussian PDF for correlations between the gaseous properties. Hartick et al. [3] applied a similar approach to study diffusion flames. TRI was again considered by using a prescribed shape for the joint PDF of the mixture fraction and chemical heat release rate. Several other researchers have attempted to predict TRI using a prescribed PDF to model the fluctuations of

the medium properties, which can predict the qualitative behavior of TRI but cannot be used for quantitative evaluation. Mazumder and Modest [4] first introduced direct PDF methods to investigate TRI in methane air diffusion flames, solving a modeled joint velocity–composition PDF equation [5, 6]. Invoking only the OTFA, no other assumptions were made regarding turbulence–radiation interactions because the PDF method solves for scalar fields and it is thus possible to extract directly calculated PDF's of the various scalars involved. A similar approach was taken by Li and Modest, except that they used the joint composition PDF method [1, 7, 8]. They also studied methane–air diffusion flames and well documented experimental flames like the Sandia-D, Sydney-L flames, etc. A detailed analysis was done on effects of various factors like Reynolds number, Damköhler number and Froude number on TRI. They reported increased fluxes by up to 50% by considering emission TRI.

Most of the work on TRI to date has used the OTFA proposed by Kabashnikov and Myasnikova [9], in which it is assumed that the turbulent eddies are optically thin, so that the fluctuations in the local medium properties are uncorrelated from fluctuations in local intensity, which implies that the absorption TRI can be reasonably neglected. The validity of this assumption would depend on the eddy size distribution and the radiative properties of absorbing gases and would generally be valid if the optical thickness of a turbulent structure is $\kappa l \ll 1$ where κ is the local absorption coefficient and l is the turbulent eddy length scale. While this assumption is reasonable for small optical thicknesses, its applicability in optically thick regions is questionable. Hartick et al. [3] have shown that OTFA is not valid in some highly absorbing parts of the water vapor spectrum though the overall effect on the spectrally integrated heat transfer is very small. Tessé et al. [10] discuss a similar situation for carbon dioxide.

It is expected that a photon Monte Carlo method can potentially account for all turbulence–radiation interactions including absorption TRI. A first attempt in that direction was made by Tessé et al. [10], who calculated radiative transfer for a sooty, turbulent ethylene–air diffusion jet flame using a photon Monte Carlo method together with a correlated-k (CK) model for the gas properties. In their TRI model the radiative properties of the assumed homogeneous turbulent structures are randomly obtained from a multidimensional PDF of the reaction progress variable, mixture ratio and the soot volume fraction and concluded that TRI yielded an increase of about 30% in radiative heat loss. The main limitation of their approach was that the turbulence structures were modeled using ad-hoc arguments. The degree of approximation introduced in modeling turbulence–radiation interactions due to OTFA remains unknown. While the photon Monte Carlo method can overcome the limitations of OTFA, it does so at great additional computational cost. The aim of this paper is to provide a simple model for absorption TRI, thus overcoming the limitations of OTFA.

2. Mathematical Formulation

Turbulence-Radiation Coupling The radiative transfer equation (RTE) for a nonscattering medium is [11]:

$$\frac{\partial I_\eta}{\partial s} = \kappa_\eta (I_{b\eta} - I_\eta), \tag{1}$$

where I_η is the spectral radiative intensity, κ_η is the spectral absorption coefficient and $I_{b\eta}$ is the Planck function. If the spectral absorption coefficient, Planck function and spectral intensity are decomposed into a mean and a fluctuating part followed by averaging yields

$$\frac{\partial \langle I_\eta \rangle}{\partial s} = \langle \kappa_\eta \rangle \langle I_{b\eta} \rangle - \langle \kappa_\eta \rangle \langle I_\eta \rangle + \left\langle \kappa'_\eta I'_{b\eta} \right\rangle - \left\langle \kappa'_\eta I'_\eta \right\rangle, \tag{2}$$

where $\langle\rangle$ denotes ensemble averages and primes denote fluctuations. The last two terms in Eq. (2) are the additional terms that arise due to the interaction between radiation and turbulence. For closure these terms need to be evaluated. The term $\left\langle \kappa'_\eta I'_{b\eta} \right\rangle$ is loosely called emission TRI and has been modeled accurately using composition PDF methods and photon Monte Carlo methods. The term $\left\langle \kappa'_\eta I'_\eta \right\rangle$ is the absorption TRI term, which is of special interest in the present paper.

Radiation Models Equation (1) is a five-dimensional PDE (3 space plus 2 direction coordinates), and is difficult to solve even if the TRI terms were known. In the current work the lowest order spherical harmonics method P_1-approximation is used, which converts the RTE into a relatively simple Helmholtz equation. Following the development of [11] one obtains for nonscattering media

$$\nabla \cdot q_\eta = \nabla \cdot \left(-\frac{1}{3\kappa_\eta} \nabla G_\eta \right) = \kappa_\eta (4\pi I_{b\eta} - G_\eta), \tag{3}$$

where incident radiation G_η is the intensity integrated over all solid angles (or all directions) given by $G_\eta = \int_{4\pi} I_\eta d\Omega$ and the boundary condition for Eq. (3) is given by,

$$-\frac{2-\epsilon}{\epsilon} \frac{2}{3\kappa_\eta} \widehat{n} \cdot \nabla G_\eta + G_\eta = 4\pi I_{bw\eta}, \tag{4}$$

where ϵ is the wall emittance and I_{bw} is the Planck function evaluated at the wall temperature.

For an optically thick nonscattering medium, the heat flux, reduces to [11]

$$\overline{q}_{\eta \text{Diffusion}} = -\frac{1}{3\kappa_\eta} \nabla [4\pi I_{b\eta}], \tag{5}$$

which is known as the diffusion approximation, implying that, as the local optical thickness increases, the incident radiation $G_\eta \rightarrow 4\pi I_{b\eta}$. One important feature of the diffusion approximation is that it depends only on local scalars and hence can, in principle be evaluated accurately using composition PDF methods.

The assumption of a gray medium almost always fails when the medium contains absorbing-emitting gases like water vapor and carbon dioxide, due to their strong spectral dependence of radiation properties. Global methods such as Weighted-Sum-of-Gray-Gases Model (WSGG) are commonly used [12] . Recently, Modest and Zhang [13] developed a Full Spectrum k-Distribution (FSK) method, which was shown to be superior to the WSGG model. The P_1-approximation and diffusion approximation will be used in conjunction with the FSK model

in this study. In FSK the spectral dependence of radiative quantities is transformed to a g-dependence, where g is a cumulative distribution function of the absorption coefficient calculated over the entire spectrum and weighted by the Planck function. Within the P_1-approximation the modified equation considering FSK for nonscattering media can be written as [13]

$$\nabla \cdot \left(-\frac{1}{3k_g} \nabla G_g \right) = 4\pi k_g a_g I_b - k_g G_g \tag{6}$$

subject to the boundary condition

$$-\frac{2-\epsilon}{\epsilon}\frac{2}{3}\widehat{n} \cdot \nabla G_g = k_g(4\pi a_{gw} I_{bw} - G_g). \tag{7}$$

Here k_g is the absorption coefficient reordered in the g-space and ϵ is the wall emittance and $[a_g I_b]$ is a weighted Planck function with a_g describing how the nongrayness varies spatially within a nonhomogeneous medium i.e., it represents stretching of the k-g distributions due to nonhomogeneities. These equations are solved for all the g-values followed by numerical quadrature. The corresponding equation for the diffusion approximation in g-space can be written as

$$\overline{q}_{g\text{Diffusion}} = -\frac{1}{3k_g} \nabla [4\pi a_g I_b]. \tag{8}$$

Composition PDF Method The basic idea of the PDF approach is to treat species concentrations and temperature as random variables and solve for a joint PDF instead of solving for the mean and the first few moments. Once the PDF is known, the mean and any required higher order moments of any quantity can be obtained as long as it is a function of the local composition and temperature only. For example, the emission TRI terms in Eq. (2) can be obtained as

$$\langle \kappa_\eta I_{b\eta} \rangle \, (\underline{\phi}) = \int \kappa_\eta(\underline{\psi}) I_{b\eta}(\underline{\psi}) f(\underline{\psi}) d\underline{\psi}, \tag{9}$$

where $\underline{\psi}$ denotes the composition space variable, $\underline{\psi} \equiv (\psi_1, \psi_2, \ldots, \psi_s)$ and $f(\underline{\psi})$ is defined to be the probability density of a joint event $\underline{\phi} = \underline{\psi}$. In a general turbulent reacting flow, the composition PDF is also a function of space, $\underline{x}$, and time, t whose transport equation $f(\underline{\psi}, \underline{x}, t)$ can be derived from the laws of conservation [6]. The most important advantage of PDF methods over moment methods is that source terms of arbitrary complexity can be handled without any further modeling as long as they are functions of the local compostion only. Therefore, evaluation of emission TRI requires no modeling in this method. The composition PDF model is not a self-contained model and requires knowledge of the mean velocity field $\widetilde{u}$, and a turbulence model to determine both the turbulent diffusivity, to model turbulent convection of the scalars in the physical space, and a mixing rate model for molecular mixing.

Monte Carlo methods are generally employed, in which the PDF is represented by a large number of notional particles, which evolve in time and space according to a set of stochastic equations and carry all the composition variables. The PDF at any point in the domain is then

obtained as a histogram of the particle properties in sufficiently small neighborhoods in physical space. The composition PDF equation is one-point one-time scheme and statistics at any point in the domain can be accurately obtained, although no correlation information about the scalars or the scalar gradients is available. This is an important point and will be referred to later on. The particle tracing algorithm by Li and Modest [14] is used.

Absorption TRI Model The composition PDF method allows accurate closure of the emission TRI terms. The problem of modeling absorption TRI is considerably more complicated due to the fact that the intensity I_η (or incident radiation G_η) are not functions of the local composition variables only, i.e., they are not transported scalars. This precludes the use of composition PDF methods or even the moment methods to obtain closure. As discussed earlier, the absorption TRI are important only if the optical thickness of an individual turbulent eddy is sufficiently large. In combustion gases such high absorption coefficients are generally limited to only a small part of the spectrum, but can be prevalent in sooty flames. The term $\langle \kappa' G' \rangle$ appears directly in the energy equation and when the diffusion approximation is valid, one can use the diffusion aproximation to solve the RTE (locally) and use any RTE solver including the P_1-approximation elsewhere.

To model absorption TRI it will be assumed that, if the diffusion approximation is valid in a certain region when no TRI are considered, then it remains a reasonable approximation when TRI are taken into account providing a net TRI, i.e., both emission as well as the absorption TRI. Other methods, such as the P_1-approximation, can only predict the emission TRI, and the difference between the two solutions would be the absorption TRI:

$$\begin{array}{rcll} \left\langle \kappa'_\eta G'_\eta \right\rangle = & \nabla \cdot \left\langle \overline{q}_\eta \right\rangle - \nabla \cdot \left\langle \overline{q}_{\eta \mathrm{Diffusion}} \right\rangle, & \mathrm{OTFA} \quad \mathrm{questionable}, \\ = & 0, & \mathrm{Otherwise}. \end{array} \tag{10}$$

In the context of the P_1-approximation and FSK methodology one obtains

$$\begin{array}{rcll} \left\langle k' G' \right\rangle_g = & \langle k_g (4\pi a_g I_b) \rangle - \langle k_g \rangle \langle G_g \rangle - \nabla \cdot \left\langle \overline{q}_{g \mathrm{Diffusion}} \right\rangle, & \mathrm{OTFA} \quad \mathrm{questionable}, \\ = & 0, & \mathrm{Otherwise}. \end{array} \tag{11}$$

One-point, one-time composition PDFs can accurately model scalars like $\langle k_g \rangle$ and combinations, such as $\langle 4\pi a_g I_b \rangle$, but they contain no information about the gradients of scalars. Therefore, the mean radiative source term predicted by the diffusion approximation cannot be extracted directly using the composition PDF method. Another difficulty is the fact that the absorption coefficient is a function of species concentrations as well as temperature and $B_g = 4\pi a_g I_b$ is a function of temperature only; hence the mean diffusion flux $-\langle 1/3 k_g \nabla B_g \rangle$ cannot be closed easily. However, this term could be closed accurately if the absorption coefficient were a function of temperature alone.

To make the absorption coefficient a function of temperature only, the radiative properties of the gas mixture will be modified slightly. This is done by reducing the actual Planck-mean absorption coefficient field of the target configuration (depending on temperature and concentrations) to a temperature-only dependent function through a least-squares fit. The assumed

Planck-mean absorption coefficient is shown in Fig. 1b compared to the actual Planck-mean absorption coefficients at all the grid nodes and larger absorption coefficient at any temperature indicates higher concentrations of $CO_2$and H_2O. Nongrayness of the gas mixture is preserved by using an artificial FSK model that recovers the modeled Planck-mean absorption coefficient, i.e.,

$$k_g = k_p(T)\frac{2}{\pi}[\tanh^{-1}(2g-1)] \tag{12}$$

The form chosen is similar to various correlations developed by Dennison et al. [15] and Modest and coworkers [16, 17], and hence this approach can in principle, be extended to these models as well. If the spectral absorption coefficient is a function of temperature only the radiative source according to the diffusion approximation becomes

$$\left\langle \overline{q}_{g\text{Diffusion}} \right\rangle = -\left\langle \frac{1}{3k_g}\nabla B_g \right\rangle = -\left\langle \frac{1}{3k_g}\frac{dB_g}{dT}\nabla T \right\rangle = -\nabla \left\langle H_g(T) \right\rangle, \tag{13}$$

where

$$H_g(T) = \int_{T_0}^{T} \frac{1}{3k_g}\frac{dB_g}{dT}dT. \tag{14}$$

Since $H_g(T)$ is a function of the local scalar T only, it can be accurately extracted and the mean diffusion heat source can be obtained at any point in the domain as the Laplacian of H_g.

Estimation of Diffusion Heat Source Mean fields in the PDF method are statistical estimates at any given point and the average scalars evaluated at any point are completely uncorrelated from those at any other point. Therefore, the averaged scalars are only C^0 continuous and cannot be differentiated, even if the scalars are time-averaged over a long time, as the resulting mean field still contains noise due to turbulent fluctuations and random statistical noise. The current focus is on evaluating the Laplacian of the function $H_g(T)$. A thin plate smoothing spline is translation and rotation invariant and hence was used to smooth the estimated mean fields to attain C^m continuity to facilitate differentiation which is the solution to a variational problem, given by

$$y_i = f(\underline{x}_i) + \epsilon_i \quad i = 1, \ldots, n \tag{15}$$

where $\epsilon = (\epsilon_1, \ldots, \epsilon_n)^T$ are random error terms and can have non-Gaussian distribution. Find f to minimize

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - f(\underline{x}_i))^2 + \lambda J_m^d(f) \tag{16}$$

The first term Eq. (16) is the loss function and the second term $J_m^d(f)$ is the roughness penalty functional of f, λ is a smoothing parameter and denotes the relative importance of the roughness penalty as compared to the conventional loss function. $J_m^d(f)$ is large if f is wiggly (or rough) and small for a smooth function. Interested reader is referred to Wahba [18] and references therein. The order of the spline m forces up to $(m-1)^{\text{th}}$ derivatives to be absolutely continuous. A value of $m = 4$ or 5 was found sufficient in the current study to get good estimates of the Laplacian. It was found that a smoothing parameters λ of 2-3 orders of magnitude higher than those obtained by Generalized Cross Validation [18] were required.

3. Results

Target Configuration An axisymmetric methane jet with co-flow of air was simulated using Hybrid finite volume/PDF Monte Carlo method. A small nozzle ($d_j = 0.01m$) at the center introduces methane at 80 m/s; ambient air enters coaxially at 0.5 m/s. The overall equivalence ratio is approximately 0.76 and Reynolds number based on jet diameter is approximately $28,000$. The PDF code is coupled closely with the finite volume code FLUENT [19] and a structured grid is used to facilitate easy evaluation of the second derivatives. A two equation standard k–ϵ model is used for the turbulence model and fast chemistry model is used, since the stress is on radiation calculations. With the fast chemistry assumption, the scalars in the problem include mass fractions of five different species (CH_4, O_2, H_2O, CO_2 and N_2) and the absolute enthalpy h of the mixture. Updated values of density and coefficients for the P_1-approximation are fed back from the PDF code to the finite volume code. Details are available in [1]. The temperature profile when only emission TRI is considered is shown in Fig. 1a. It should be noted that the peak temperature drops by about 220 K and the total radiative heat loss increases from 45 kW to 53kW when emission TRI are considered. Figure 1b shows the local Planck-mean absorption coefficient at the grid points, as well as the fit used within this study.

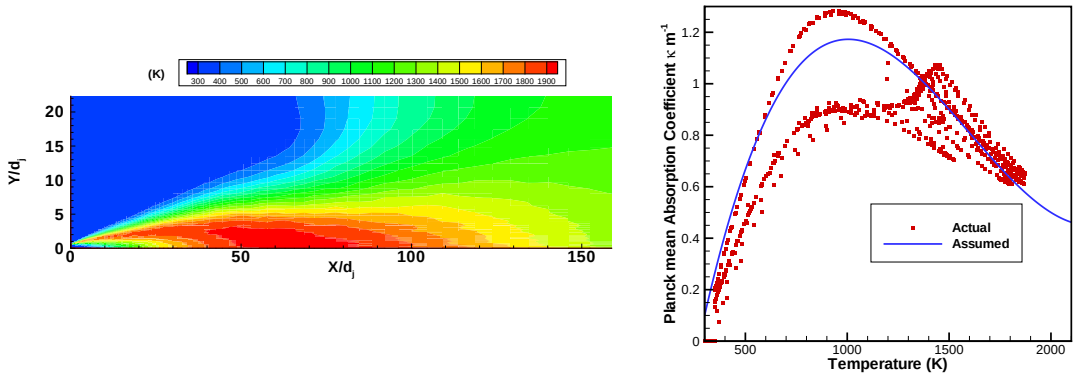

Fig. 1 a) Temperature profile of methane jet flame with emission TRI b) Assumed Planck-mean absorption coefficeint and the actual evaluated at differnt locations in the domain.

Applicability of Diffusion Approximation The local absorption coefficient varies greatly in the computational domain and, hence, it is very difficult to identify the regions where diffusion approximation is applicable. In the first step, a smooth temperature field $\langle T \rangle_s$ is obtained by using the spline model with sufficiently large smoothing parameter. The corresponding absorption coefficient field $k_g(\langle T \rangle_s)$ and the function $H_g(\langle T \rangle_s)$ are calculated. The radiative source terms obtained from P_1 and diffusion approximations are then compared to see if the diffusion approximation is valid. It was found that as the absorption coefficient increased, the diffusion approximation became valid over most of the domain excluding the low temperature regions where optical thickness was smaller and neither emission nor absorption TRI are of importance. The pseudo-spectral heat source (calculated at the largest g-value) using the two methods for

various axial locations is shown in Fig. 2 which shows that the diffusion approximation is in close agreement with the P_1 approximation. The maximum optical thickness in this case, based on the cylinder radius is $(k_g)_{\max} L \sim 175$. For lower optical thicknesses it was found that the diffusion approximation was relatively inaccurate in a larger part of the domain and was generally inaccurate when local optical thickness was below $k_g L < 80$. The total pseudo-spectral radiative heat loss predicted by the effective radiative source using the diffusion approximation in optically thick regions and corresponding P_1-approximation in optically thinner regions was found to be within 2.5% of the local heat loss predicted using the P_1approximation almost everywhere.

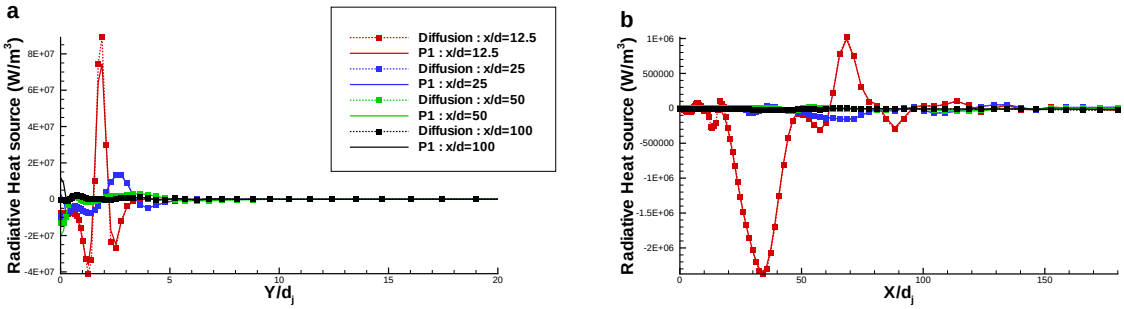

Fig. 2 Pseudo-spectral (i.e. at the highest g-value) radiative heat sources at a) constant axial b) radial locations with no TRI using the $P_1$and diffusion approximations .

It can be seen that near the inlet where there are very high temperature gradients, the diffusion approximation is valid only up to a small radial distance since most of the cross section is filled with unreacted low temperature gas with essentially no absorption. As the axial distance increases, the diffusion approximation becomes valid for a larger part of the domain and beyond $x/d_j = 80$ it was observed that the diffusion approximation was valid all the way to the boundary. In all this region it is expected that, when the calculations are done with the TRI model turned on, the diffusion approximation would still be applicable and the difference between the two heat sources would be the absorption TRI.

Investigation of Absorption TRI Absorption TRI are studied without considering feedback to the scalar field, i.e., the mean scalar fields, heat fluxes and the turbulent moments of quantities involved in radiation are calculated from a frozen particle field obtained from the PDF solution. When TRI are to be determined, one needs to solve the P_1 equation and also estimate the diffusion heat source. To obtain good estimates of these two radiative sources, three variables are smoothed, $\langle k_g a_g I_b \rangle_s$, $\langle k_g \rangle_s$ and $\langle H_g(T) \rangle_s$ where the subscript s indicates sufficiently smoothed scalars. Following this development, the corresponding radiative heat source terms including the TRI, as obtained by the P_1 and diffusion approximations for the given reordered spectral variable g are shown in Fig. 3a and 3b. The difference between the two heat sources is shown in Fig. 3c for the largest pseudo-spectral g-value used in the spectral interation of Eq. (3) this is the absorption TRI as defined by Eq. (11). An effective radiative source term includes the modeled absorption TRI and the emission TRI obtained through the PDF method. As seen from Fig. 3,

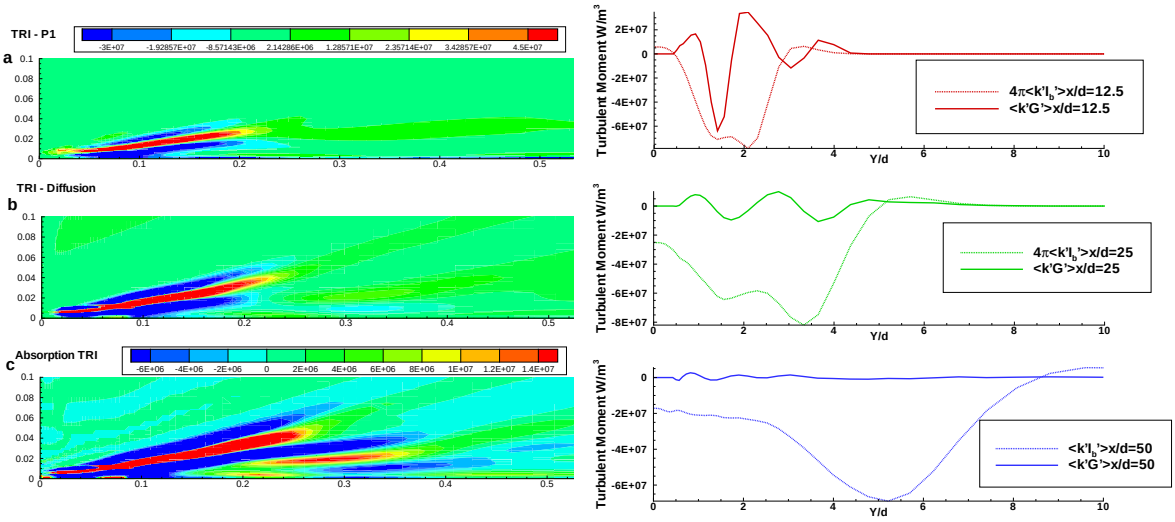

Fig. 3 a) P_1 approximation b) Diffusion approximation c) Difference between the two which is also the pseudo-spectral (i.e. at the higest g-value) absorption TRI d) Comparison of radial emission TRI and absorption TRI values at given axial location

absorption TRI has positive as well as negative regions. It is interesting to note that, in this case, the volume integrated absorption TRI is essentially zer, i..e, $\int \langle k'G' \rangle \, dV / \int \nabla \cdot q_{P_1} dV \simeq 0.2\%$.

In order to obtain more insight into absorption TRI, one needs to examine its local behavior. As the optical thickness increases $G \rightarrow 4\pi I_b$, so that $\left\langle k'_g G'_g \right\rangle \rightarrow \left\langle 4\pi k'_g a'_g I'_b \right\rangle$. The emission TRI are directly related to turbulent fluctuations of the scalars, whereas the absorption TRI are more dependent on gradients of the scalars according to the formulation used. *Figure* 3d shows both absorption TRI and emission TRI for different axial locations. As one proceeds in the axial direction, the local gradients reduce significantly thereby reducing the effects of absorption TRI. However, the fluctuations are only slightly reduced as there is turbulent mixing and emission TRI are not much affected. It can be seen that near the mixing layer where there are very high gradients, the absorption TRI is significant and comparable to emission TRI and tend to reduce the net TRI as expected even for the high optical thickness considered here, the absorption TRI are not important beyond the flame outside of the active reaction zone. Finally, for the gas mixture considered here, absorption TRI ocur only for the narrow, optically thcinest regions (large g-values), with small impact on the total spectrum-integrated fluxes. Howerer, for highly sooting flames these effects of absorption TRI are expected to be important over large parts of the flame.

4. Summary

Absorption TRI was investigated and it was found that it significantly changes the spectral heat loss from optically thick turbulent eddies. The most significant differences were found to be in the flame sheet region, which contain very strong gradient regions for quantities involved in radiation. However, absorption TRI are positive in some regions and negative in others, so

that the overall radiative heat loss may not be significantly affected. It is expected that the absorption TRI could be of great importance for turbulent sooty flames, which have optically thick conditions over much of the spectrum. This work provides a tool to investigate absorption TRI in large sooting/nonsooting flames, where the absorption coefficients are expected to be much larger over a large parts of the spectrum.

Acknowledgement This work has been supported by the National Science Foundation under Grant Number CTS-0121573.

REFERENCES

[1] LI, G. and MODEST, M. F., Importance of Turbulence–Radiation Interactions in Turbulent Diffusion Jet Flames, *ASME J. Heat Transfer*, 125, 831–838, 2003.

[2] SONG, T. and VISKANTA, R., Interaction of Radiation with Turbulence: Application to a Combustion System, *J. Thermoph. Heat Transfer*, 1, 56–62, 1987.

[3] HARTICK, J. W., TACKE, M., FRUCHTEL, G., HASSEL, E. P., and JANICKA, J., Interaction of Turbulence and Radiation in Confined Diffusion Flames, in *Twenty-Sixth Symposium (International) on Combustion*, The Combustion Institute, 75–82, 1996.

[4] MAZUMDER, S. and MODEST, M. F., PDF Modeling of Turbulence Radiation Interactions, in *1997 National Heat Transfer Conference, Baltimore, MD*, ASME, 1997.

[5] POPE, S. B., *Turbulent Flows*, Cambridge University Press, Cambridge, 2000.

[6] POPE, S. B., PDF Methods for Turbulent Reactive Flows, *Progr. Energy Combust. Sci.*, 11, 119–192, 1985.

[7] LI, G. and MODEST, M. F., Application of composition PDF methods in the investigation of turbulence–radiation interactions, *JQSRT*, 73, 461–472, 2002.

[8] LI, G. and MODEST, M. F., Numerical simulation of turbulence-radiation interactions in turbulent reacting flows, in SUNDEN, B. and FAGHRI, M., eds., *Modelling and Simulation of Turbulent Heat Transfer*, WIT Press, Southampton, England, 2004.

[9] KABASHNIKOV, V. P. and MYASNIKOVA, G. I., Thermal Radiation in Turbulent Flows—Temperature and Concentration Fluctuations, *Heat Transfer-Soviet Research*, 17(6), 116–125, 1985.

[10] TESSÉ, L., DUPOIRIEUX, F., and TAINE, J., Monte Carlo modeling of radiative transfer in a turbulent sooty flame, *Intl. J. Heat Mass Transfer*, 47, 555–572, 2004.

[11] MODEST, M. F., *Radiative Heat Transfer*, Academic Press, New York, 2nd edn., 2003.

[12] DENISON, M. K. and WEBB, B. W., k-Distributions and Weighted-Sum-of-Gray Gases: A Hybrid Model, in *Tenth International Heat Transfer Conference*, Taylor & Francis, 19–24, 1994.

[13] MODEST, M. F. and ZHANG, H., The Full-Spectrum Correlated-*k* Distribution For Thermal Radiation from Molecular Gas–Particulate Mixtures, *ASME J. Heat Transfer*, 124(1), 30–38, 2002.

[14] LI, G. and MODEST, M. F., An Effective Particle Tracing Scheme on Structured/Unstructured Grids in Hybrid Finite Volume/PDF Monte Carlo Methods, *J. Comp. Phys.*, 173, 187–207, 2001.

[15] DENISON, M. K. and FIVELAND, W. A., A Correlation for the Reordered Wave Number of the Wideband Absorptance of Radiating Gases, *ASME J. Heat Transfer*, 119, 853–856, 1997.

[16] MODEST, M. F. and MEHTA, R. S., Full spectrum k-distribution correlations for CO_2 from the CDSD-1000 spectroscopic databank, *Intl. J. Heat Mass Transfer*, 47, 2487–2491, 2004.

[17] MODEST, M. F. and SINGH, V., Engineering correlations for full spectrum k-distribution of H_2O from the HITEMP spectroscopic databank, *JQSRT*, 93, 263–271, 2005.

[18] WAHBA, G., *Spline Models for Observational Data*, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1990.

[19] FLUENT, *FLUENT 6.0 UDF Manual*, Fluent Inc., New Hampshire, 2001.