The Greenhouse Effect

The Radiative Flux (W m^{-2}) for a blackbody at temperature $T$ is given by

$$F(T) = \sigma T^4$$

where the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

The area of a sphere of radius $R$ is $4\pi R^2$

The area of a circle of radius $R$ is $\pi R^2$

![Figure: Solar radiation spectra measured from a satellite outside Earth’s atmosphere (in bold) and at sea level.](image)

1. Does the Sun approximate a blackbody? Yes. The solar radiation spectrum measured outside the Earth’s atmosphere approximates an idealized black body spectrum for a 5800K object.

2. What percent of incoming radiation is absorbed by a blackbody? 100%... that is the definition of a blackbody.

3. The temperature of the Sun is 5800 K. What is the radiation flux at the surface of the Sun in W m^{-2}? $\sigma T^4 = 6.4 \times 10^7$ W m^{-2}

4. The radius of the sun is $7 \times 10^8$ m. What is the total radiation emitted by the Sun over its entire surface in Watts (W)? $\text{Area}(R) \cdot \sigma T^4 = 4\pi R^2 \cdot \sigma T^4 = 4\pi R^2 \cdot 6.4 \times 10^7 = 3.94 \times 10^{26}$ W
5. The Earth is a distance \( d = 1.5 \times 10^{11} \text{ m} \) from the Sun. At that distance, the solar radiation flux \( F_s \) is distributed uniformly over a sphere centered on the Sun with radius \( d \). What is the solar radiation flux \( F_s \) in \( \text{W m}^{-2} \)?

\[
(Area(R)\sigma T^4) / (Area(d)\sigma T^4) = 3.94 \times 10^{26} / 4\pi d^2 = 1394 \text{ W m}^{-2}
\]

6. Assume the Earth has an albedo of 0, so that all the solar energy intercepted by the Earth is absorbed. What fraction of the solar radiation flux \( F_s \) is absorbed on average per unit area of the Earth’s surface?

Cross-section area / surface area = \( \pi r^2 / 4\pi r^2 = 1/4 \)

7. Suppose the Earth is a perfect cube with each edge the same length, and the same side always faces the sun (i.e., it doesn’t rotate). What fraction of the solar radiation flux would be absorbed on average per unit area of the Earth’s surface?

Cross-section area / surface area = \( r^2 / 6r^2 = 1/6 \)

8. In equilibrium, at each moment in time the Earth radiates as much energy as it absorbs from the Sun. If the albedo is 0 and there is no atmosphere, what is the surface temperature of the Earth? (assume the Earth is a sphere).

\[ 1394/4 = \sigma T^4 \rightarrow 280 \text{ K} \]
9. Suppose the Earth reflects 28% of intercepted radiation (its albedo is 0.28). If there is no atmosphere, what is the radiative flux absorbed by the Earth’s surface in $\text{W m}^{-2}$?

$$\text{Percent Absorbed} \times \text{Solar Flux} = \left(\frac{1}{4} \times (1-A)\right) \times 1394 = \frac{1}{4} \times 0.72 \times 1394 = 251 \text{ W m}^{-2}$$

10. If there is no atmosphere, what is the surface temperature of the Earth?

$$\sigma T^4 = 251 \text{ W m}^{-2} \Rightarrow T = 258 \text{ K}$$
Now assume there is a single layer of atmosphere above the surface of the Earth that is transparent to solar radiation, but absorbs a fraction $f$ of terrestrial radiation due to the presence of greenhouse gases.

The Earth’s surface has temperature $T_0$ and the atmospheric layer has a temperature $T_1$.

To an observer in space, the radiative flux from the top of the atmosphere is given by:

$$(1-f)F(T_0) + fF(T_1)$$

11. Briefly describe in words what $(1-f)F(T_0)$ represents.

Radiation emitted from the surface that passes through the Atmosphere without being absorbed.

12. Briefly describe in words what $fF(T_1)$ represents.

Radiation reemitted upwards from the Atmosphere.

13. In equilibrium $f F(T_0) = 2 f F(T_1)$. Why?

Atmospheric layer emits $f F(T_1)$ up and down. Sum of these must equal the amount absorbed: $f F(T_0)$

14. If $f = 0.77$ and the Earth + Atmosphere system is in equilibrium, what are the values of $T_0$ and $T_1$?

$$f F(T_1) = 1/2 f F(T_0) \Rightarrow (1-f)F(T_0) + fF(T_1) \Rightarrow (1-f)F(T_0) + f/2 F(T_0) \Rightarrow (1-0.77/2) F(T_0)$$

emitted from top of the Atmosphere. In Equilibrium: $251 = (1 - f/2) \sigma T_0^4 \Rightarrow$

$$T_0 = \frac{251}{(1 - 0.385)\sigma} = 291 K$$

$$f F(T_1) = 1/2 f F(T_0) \Rightarrow T_1 = T_0 / 2^{1/4} = 245 K$$

15. If we increase the concentration of greenhouse gasses in the atmosphere such that $f$ increases to 0.8, what are the new $T_0'$ and $T_1'$ once a new equilibrium is achieved?

$$T_0' = \frac{251}{(1 - 0.8)\sigma} = 293 K$$

$$T_1' = T_0' / 2^{1/4} = 246 K$$
Suppose we tried to reduce the rate at which greenhouse gasses accumulate in the atmosphere by building solar power satellites that convert light from the sun to electricity in orbit around the Earth, and then beamed this energy to microwave (wavelength = 12cm) receptor stations on the planet. This could potentially reduce greenhouse gas emissions if it replaced energy generation that uses carbon-based fuels.

16. Draw a picture of a layer model with one atmospheric layer and the satellites added in, and write down an energy budget for the surface, the atmospheric layer, and for the Earth at the top of the atmosphere as observed from space.

Results here depend on your assumptions, which need to be clearly stated up front.

17. What would the direct effect of this system be on the surface temperature of the Earth, independent of any effects it might have on greenhouse gas emissions. (i.e, would the surface temperature stay the same, increase or decrease)?

Recall that in class during the exam I pointed out that in answering this question, you should extrapolate from your layer model in question 16 to a sphere. Under reasonable assumptions, this system of power generation is likely to warm the earth. Why?
18. Consider the following two spectrum graphs.
What has changed between the first and second graph?

*The bands around wave number 1300 are now saturated due to an increase in atmospheric methane concentration. The apparent temperature around this wave number is lower because more of the energy emitted from the surface is absorbed and re-emitted by the atmosphere, which is cooler than the surface. Elsewhere, intensities have increased on average, consistent with a warmer surface temperature.*